

$ud\bar{b}\bar{b}$ tetraquark resonances with lattice QCD potentials and the Born-Oppenheimer approximation

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We study tetraquark resonances with lattice QCD potentials computed for a static $\bar{b}\bar{b}$ pair in the presence of two lighter quarks ud , the Born-Oppenheimer approximation and the emergent wave method. As a proof of concept we focus on the system with isospin $I = 0$, but consider different relative angular momenta l of the heavy quarks $\bar{b}\bar{b}$. For $l = 0$ a bound state has already been predicted with quantum numbers $I(J^P) = 0(1^+)$. Exploring various angular momenta we now compute the phase shifts and search for S and T matrix poles in the second Riemann sheet. We predict a tetraquark resonance for $l = 1$, decaying into two B mesons, with quantum numbers $I(J^P) = 0(1^-)$, mass $m = 10\,576^{+4}_{-4}$ MeV and decay width $\Gamma = 112^{+90}_{-103}$ MeV.

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I. INTRODUCTION

A long standing problem in QCD is to understand exotic hadrons, i.e. hadrons which have a structure more complicated than a quark-antiquark pair or a triplet of quarks [1]. The problem of identifying exotic hadrons, say tetraquarks, pentaquarks, hexaquarks, hybrids or glueballs, turned out to be much harder than initially expected [2]. The observed candidates are resonances high in the spectrum, not only difficult to observe, but also technical to address in hadronic models and possibly requiring new technical developments to be studied theoretically from first principles e.g. with lattice QCD [3, 4].

Our main motivation is to investigate tetraquarks by combining lattice QCD and quantum mechanics techniques. We specialize in systems with two heavy antiquarks, which are expected to form bound states, when sufficiently heavy [5–16]. The starting point are potentials of two static antiquarks in the presence of two light quarks, which can be computed with state of the art lattice QCD techniques (cf. e.g. [17–22]). If the masses of the two heavy quarks are much larger than the scale of QCD, which is the case for two \bar{b} quarks, their dynamics can then be described by a quantum mechanical Hamiltonian with the aforementioned lattice QCD potentials. This two-step approach is the Born-Oppenheimer approximation [23]. Using this approach, a $ud\bar{b}\bar{b}$ tetraquark bound state with quantum numbers $I(J^P) = 0(1^+)$ has recently been predicted [21, 22, 24–26] and confirmed by a lattice QCD computation with four quarks of finite mass

[27]. So far, however, resonances have not been studied in this framework.

In this work we extend the previous Born-Oppenheimer studies with lattice QCD potentials, reviewed in Section II. We utilize the emergent wave method, a technique from scattering theory detailed in Section III, to compute phase shifts, S and T matrix poles in the second Riemann sheet and the corresponding resonance masses and decay widths. Our results are presented in Section IV. We conclude in Section V.

II. LATTICE QCD POTENTIALS OF TWO STATIC ANTIQUARKS IN THE PRESENCE OF TWO LIGHT QUARKS

Potentials $V(r)$ of two static antiquarks $\bar{Q}\bar{Q}$ in the presence of two light quarks qq have been computed using lattice QCD for many different quantum numbers including light flavor combinations qq with $q \in \{u, d, s, c\}$, parity P and light total angular momentum j (cf. e.g. [22, 25]). There are both attractive and repulsive channels. Most promising with respect to the existence of tetraquark bound states or resonances are light quarks $q \in \{u, d\}$ together with $(I = 0, j = 0)$ or $(I = 1, j = 1)$, where I denotes isospin, since the corresponding potentials $V(r)$ are not only attractive, but also rather wide and deep.

The lattice QCD results for these two potentials can be parameterized by a screened Coulomb potential,

$$V(r) = -\frac{\alpha}{r} e^{-r^2/d^2}. \quad (1)$$

This ansatz is inspired by one-gluon exchange at small $\bar{Q}\bar{Q}$ separations r and a screening of the Coulomb potential due to the formation of two B mesons at large r , as illustrated in Fig. 1. The values of the two parameters α and d as determined in [22] are listed in Table I. Clearly,

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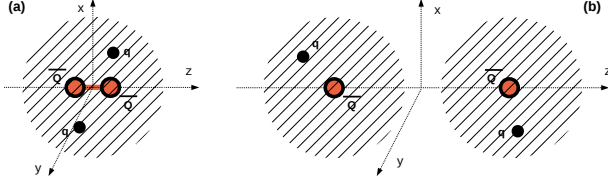


Figure 1. (Colour online.) (a) At small separations the static antiquarks $\bar{Q}\bar{Q}$ interact by perturbative one-gluon exchange. (b) At large separations the light quarks qq screen the interaction and the four quarks form two rather weakly interacting B mesons.

the $(I = 0, j = 0)$ potential is more attractive than the $(I = 1, j = 1)$ potential.

Applying the Born-Oppenheimer approximation, Eq. (1) is used as a potential for two heavy antiquarks, i.e. $\bar{b}\bar{b}$, in the presence of two light quarks ud or for two heavy-light mesons, i.e. $B^{(*)}B^{(*)}$. Solving the Schrödinger equation for the $(I = 0, j = 0)$ potential and angular momentum $l = 0$ of the two \bar{b} quarks a bound state has been predicted with binding energy 90^{+43}_{-36} MeV [22].

III. THE EMERGENT WAVE METHOD

We now summarize the emergent wave method, explained in detail for instance in Ref. [2], which is suited to study phase shifts and resonances. Let us consider the same Schrödinger equation utilized in the bound state study,

$$(H_0 + V(r))\Psi = E\Psi. \quad (2)$$

The first step is to split the wave function into two parts,

$$\Psi = \Psi_0 + X, \quad (3)$$

where Ψ_0 is the incident wave, a solution of the free Schrödinger equation,

$$H_0\Psi_0 = E\Psi_0, \quad (4)$$

and X is the emergent wave. Substituting Eq. (3) into Eq. (2) and using Eq. (4) we obtain

$$(H_0 + V(r) - E)X = -V(r)\Psi_0. \quad (5)$$

For any energy E we can use this equation to calculate the emergent wave X by providing the corresponding Ψ_0 and fixing the appropriate boundary conditions. From the asymptotic behaviour of X we then determine the phase shifts, the S matrix and the T matrix.

The problem can be continued to complex energies in a straightforward way and we can, therefore, find the poles of the S matrix and the T matrix in the complex plane. We identify a resonance with a pole, when located in the second Riemann sheet at $m - i\Gamma/2$, where m is the mass and Γ is the decay width of the resonance.

I	j	α	d in fm
0	0	$0.34^{+0.03}_{-0.03}$	$0.45^{+0.12}_{-0.10}$
1	1	$0.29^{+0.05}_{-0.06}$	$0.16^{+0.05}_{-0.02}$

Table I. Parameters α and d of the potential of Eq. (1) for two static antiquarks $\bar{Q}\bar{Q}$, in the presence of two light quarks qq with quantum numbers I and j , as determined in [22].

A. Partial wave decomposition

The Hamiltonian describing the two heavy antiquarks $\bar{b}\bar{b}$ at vanishing total momentum, i.e. in the rest frame of the system, is

$$H = H_0 + V(r) = -\frac{\hbar^2}{2\mu}\Delta + V(r) \quad (6)$$

with reduced mass $\mu = M/2$, where $M = 5280$ MeV is the mass of the B meson from the PDG [28]. For simplicity we omit the additive constant $2M$ in Eq. (6), i.e. all resulting energy eigenvalues are energy differences with respect to $2M$. We consider an incident plane wave $\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}}$, which can be expressed as a sum of spherical waves,

$$\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_l (2l+1)i^l j_l(kr) P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}), \quad (7)$$

where j_l are spherical Bessel functions, P_l are Legendre polynomials and the relation between energy and momentum is $\hbar k = \sqrt{2\mu E}$. For a spherically symmetric potential $V(r)$ as in Eq. (1) and an incident wave $\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}}$ the emergent wave X can also be expanded in terms of Legendre polynomials P_l ,

$$X = \sum_l (2l+1)i^l \frac{\chi_l(r)}{kr} P_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}). \quad (8)$$

Inserting Eq. (7) and Eq. (8) into Eq. (5) leads to a set of ordinary differential equations for χ_l ,

$$\left(-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)}{2\mu r^2} + V(r) - E \right) \chi_l(r) = -V(r)kr j_l(kr). \quad (9)$$

B. Solving the differential equations for the emergent wave

The potentials $V(r)$, Eq. (1), are exponentially screened, i.e. $V(r) \approx 0$ for $r \geq R$, where $R \gg d$. For large separations $r \geq R$ the emergent wave is, hence, a superposition of outgoing spherical waves, i.e.

$$\frac{\chi_l(r)}{kr} = i t_l h_l^{(1)}(kr), \quad (10)$$

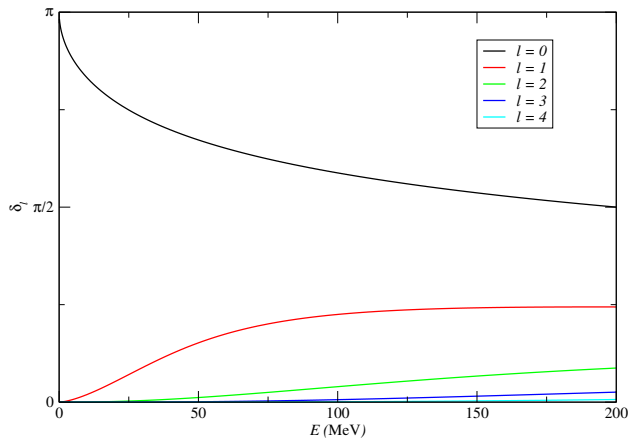


Figure 2. (Colour online.) Phase shift δ_l as a function of the energy E for different angular momenta $l = 0, 1, 2, 3, 4$ for the $(I = 0, j = 0)$ potential ($\alpha = 0.34$, $d = 0.45$ fm).

where $h_l^{(1)}$ are the spherical Hankel functions of first kind.

Our aim is now to compute the complex prefactors t_l , which will eventually lead to the phase shifts. To this end we solve the ordinary differential equation (9). The corresponding boundary conditions are the following:

- At $r = 0$: $\chi_l(r) \propto r^{l+1}$.
- For $r \geq R$: Eq. (10).

Note that the boundary condition for $r \geq R$ depends on t_l . For a given value of the energy E this boundary condition is only fulfilled for a specific corresponding value of t_l . In other words the boundary condition for $r \geq R$ fixes t_l as a function of E .

The numerical solution of the differential Eq. (9) is rather straightforward. To check our results and to exclude any numerical artefacts we implemented two different approaches: (1) a fine uniform discretization of the interval $[0, R]$, which reduces the differential equation to a large set of linear equations, which can be solved rather efficiently, since the corresponding matrix is tridiagonal; (2) a standard 4-th order Runge-Kutta shooting method.

C. Phase shifts and S and T matrix poles

The quantity t_l is a T matrix eigenvalue (cf. standard textbooks on quantum mechanics and scattering, e.g. [29]). From t_l we can calculate the phase shift δ_l and also read off the corresponding S matrix eigenvalue s_l [30],

$$s_l \equiv 1 + 2it_l = e^{2i\delta_l}. \quad (11)$$

Moreover, note that both the S matrix and the T matrix are analytical in the complex plane. They are well-defined for complex energies E . Thus, our numerical method can as well be applied to solve the differential Eq. (9) for complex E . We find the S and T matrix poles by

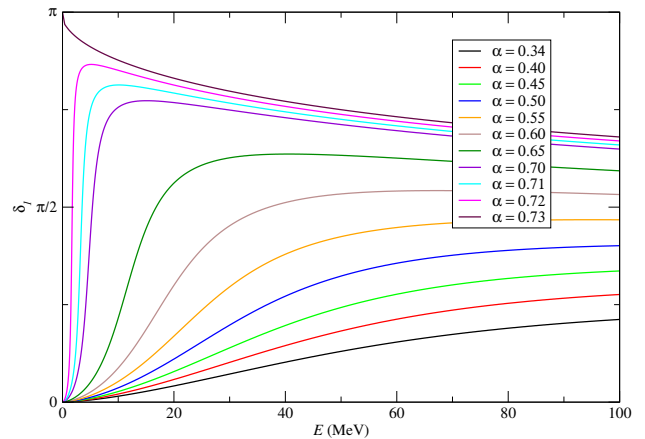


Figure 3. (Colour online.) Phase shift δ_1 as a function of the energy E for different parameters α for the $(I = 0, j = 0)$ potential ($d = 0.45$ fm).

scanning the complex plane ($\text{Re}(E), \text{Im}(E)$) and applying Newton's method to find the roots of $1/t_l(E)$. The poles of the S and the T matrix correspond to complex energies of resonances. Note the resonance poles must be in the second Riemann sheet with a negative imaginary part both for the energy E and the momentum k .

IV. RESULTS FOR PHASE SHIFTS, S MATRIX AND T MATRIX POLES AND RESONANCES

We first consider the more attractive $ud\bar{b}\bar{b}$ potential corresponding to isospin $I = 0$ and light spin $j = 0$ (cf. Sec. II). We compute t_l and via Eq. (11) the phase shift δ_l for real energy E and angular momenta $l = 0, 1, 2, \dots$. A very clear signal for a resonance would be a fast increase of the phase shift δ_l as a function of E from 0 to $\approx \pi$, almost like a step function. However, we do not find such a pronounced increase (cf. Fig. 2). Thus, we must search more thoroughly for possibly existing resonances.

Starting with angular momentum $l = 1$ we first search for clear resonance signals by making the potential more and more attractive. We increase the parameter α , while keeping the parameter $d = 0.45$ fm fixed, to preserve the scale of the potential. The corresponding results for the phase shift δ_1 are shown in Fig. 3. Indeed, for $\alpha \gtrsim 0.65$ we find clear resonances with δ_1 increasing from 0 to $\approx \pi$. Then, for $\alpha = 0.72$, we find a bound state, since the phase shift δ_1 starts at π and decreases monotonically to 0, when increasing the energy E . However, from these phase shifts it is not clear, for which values of α a resonance exists or not, i.e. it is not possible to say, whether there is a resonance for e.g. $\alpha \approx 0.50$ or even for the physical $\alpha = 0.34$.

Thus, we search directly for poles of the T matrix eigenvalues t_l . With this technique we clearly find a pole for angular momentum $l = 1$ and physical values of the parameters, $\alpha = 0.34$ and $d = 0.45$ fm. We show this pole

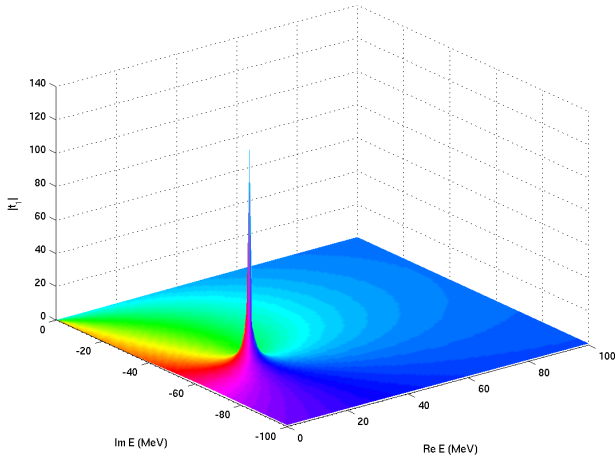


Figure 4. (Colour online.) T matrix eigenvalue t_1 as a function of the complex energy E for the $(I = 0, j = 0)$ potential ($\alpha = 0.34$, $d = 0.45$ fm). Along the vertical axis we show the norm $|t_1|$, while the phase $\arg(t_1)$ corresponds to different colours.

in Fig. 4 by plotting the t_1 as a function of the complex energy E . The pole is clearly visible as a sharp peak.

To understand the dependence of the resonance pole on the shape of the potential, we again scan different values of the parameter α and determine each time the pole of the eigenvalue t_1 of the T matrix. We show the trajectory of the pole corresponding to a variation of α in the complex plane $(\text{Re}(E), \text{Im}(E))$ in Fig. 5. Indeed, starting with $\alpha = 0.21$ we find a pole. This confirms our prediction of a resonance for angular momentum $l = 1$ and physical values of the parameters, $\alpha = 0.34$ and $d = 0.45$ fm.

Finally we perform a detailed statistical and systematic error analysis of the pole of t_1 for the physical values of the parameters, $\alpha = 0.34$ and $d = 0.45$ fm. We use the same analysis method as for our previous study of the bound state for $l = 0$, which is rather evolved and explained in detail in [25]. Our systematic errors are depicted in Fig. 5, they do not prevent the existence of a pole. With our combined statistical and systematic errors, we find a resonance energy $\text{Re}(E) = 17^{+4}_{-4}$ MeV and a decay width $\Gamma = -2\text{Im}(E) = 112^{+90}_{-103}$ MeV. Using the Pauli principle and considering the symmetry of the quarks with respect to colour, flavour, spin and their spatial wave function one can determine the quantum numbers of the resonance, which are $I(J^P) = 0(1^-)$. The resonance will decay into two B mesons and, hence, its mass is $m = 2M + \text{Re}(E) = 10576^{+4}_{-4}$ MeV.

In what concerns angular momenta $l \neq 1$, we find no clear signal for a resonance pole (except for the bound state pole for $l = 0$). We also find no poles for any l in the less attractive case of $(I = 1, j = 1)$.

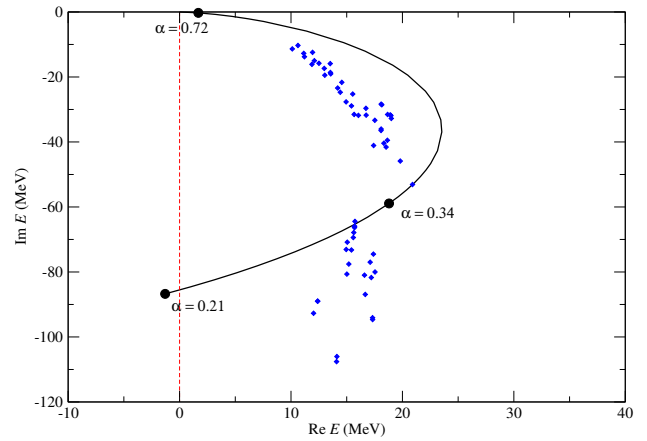


Figure 5. (Colour online) Trajectory of the pole of the eigenvalue t_1 of the T matrix in the complex plane $(\text{Re}(E), \text{Im}(E))$, corresponding to a variation of α for the $(I = 0, j = 0)$ potential ($d = 0.45$ fm). We also illustrate with a cloud of diamond points the computation of the systematic error, utilizing the technique of Ref. [25].

V. CONCLUSIONS AND OUTLOOK

As a case study for the investigation of resonances above the BB meson pair threshold, we have explored the $ud\bar{b}\bar{b}$ four-quark system. We have utilized lattice QCD potentials computed for two static antiquarks in the presence of two light quarks, the Born-Oppenheimer approximation and the emergent wave method for the BB system. First we have computed scattering phase shifts. Then we have performed the analytic continuation of the S matrix and the T matrix to the second Riemann sheet and have searched for poles as signals of resonances.

From these results we have predicted a new resonance, with quantum numbers $I(J^P) = 0(1^-)$. Performing a careful statistical and systematic error analysis has led to a resonance mass $m = 10576^{+4}_{-4}$ MeV and a decay width $\Gamma = 112^{+90}_{-103}$ MeV.

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